



INDIAN SCHOOL AL WADI AL KABIR

ASSESSMENT-1

Subject: MATHEMATICS

Class :XII Marking scheme (SET1) Max. Marks: 80

Date: 22-09-2026 Time: 3 hrs

SECTION A

1	B) 2^{20}	11. C) 12
2	A) 576	12. D) $\frac{1}{x \log x}$
3	D) $\begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$	13. D) $\sin x + \cos x + c$
4	C) 8 or -8	14. B) $\frac{x^2}{2} + \frac{\pi}{4}x + C$
5	C) $\frac{5\pi}{6}$	15. A) $[-2, \infty)$
6	B) $0 \leq y \leq 2\pi$	16. B) 1
7	B) ≥ 2	17. D) Transitive only
8	A) 5×7	18. A) $[-\sqrt{3}, -1] \cup [1, \sqrt{3}]$
9	B) $3e^{2x+4}$	19. D) (A) is false but (R) is true
10	B) 0	20. A) Both (A) and (R) are true and (R) is the correct explanation of (A)

SECTION B

21

$$A^2 = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+9 & -9-9 \\ -9-9 & 9+9 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix}$$

$$A^2 = \lambda A = \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \text{ therefore } \lambda = 6$$

1 m

1 m

22.

put $\theta = 3x$. Then

$$\frac{\cos 3x - \sin 3x}{\cos 3x + \sin 3x} = \frac{1 - \tan 3x}{1 + \tan 3x} = \tan \left(\frac{\pi}{4} - 3x \right).$$

Therefore,

$$\cot^{-1} \left[\tan \left(\frac{\pi}{4} - 3x \right) \right].$$

Now,

$$\tan \theta = \cot \left(\frac{\pi}{2} - \theta \right).$$

Hence,

$$\tan \left(\frac{\pi}{4} - 3x \right) = \cot \left(\frac{\pi}{4} + 3x \right).$$

Since $0 < x < \frac{\pi}{6}$,

$$\frac{\pi}{4} < \frac{\pi}{4} + 3x < \frac{3\pi}{4},$$

and the principal value of $\cot^{-1}$ lies in $(0, \pi)$. Thus,

$$\boxed{\cot^{-1} \left(\frac{\cos 3x - \sin 3x}{\cos 3x + \sin 3x} \right) = \frac{\pi}{4} + 3x.}$$

1 m

 $\frac{1}{2}$ m $\frac{1}{2}$ m

OR

$$\tan \left(\sin^{-1}(1) - \cos^{-1} \left(-\frac{1}{2} \right) \right)$$

$$\sin^{-1}(1) = \frac{\pi}{2}$$

$$\cos^{-1} \left(-\frac{1}{2} \right) = \frac{2\pi}{3}.$$

$$\tan \left(\frac{\pi}{2} - \frac{2\pi}{3} \right) = \tan \left(-\frac{\pi}{6} \right) = -\frac{1}{\sqrt{3}}.$$

$$\boxed{-\frac{1}{\sqrt{3}}}$$

 $\frac{1}{2}$ m $\frac{1}{2}$ m $\frac{1}{2}$ m $\frac{1}{2}$ m

23.

$$\begin{aligned}
 &\text{Let } x = \sin \theta \Rightarrow dx = \cos \theta d\theta. \\
 \therefore \int \frac{x^2}{\sqrt{1-x^2}} dx &= \int \frac{\sin^2 \theta}{\sqrt{1-\sin^2 \theta}} \cos \theta d\theta \\
 &= \int \sin^2 \theta d\theta \\
 &= \int \frac{1 - \cos 2\theta}{2} d\theta \\
 &= \frac{1}{2} \left[\theta - \frac{\sin 2\theta}{2} \right] + C \\
 &= \frac{1}{2} [\theta - \sin \theta \cos \theta] + C \\
 &= \frac{1}{2} \left[\sin^{-1} x - x\sqrt{1-x^2} \right] + C.
 \end{aligned}$$

 $\frac{1}{2}$ m $\frac{1}{2}$ m $\frac{1}{2}$ m $\frac{1}{2}$ m

24.

Given: $\frac{dV}{dt} = -1 \text{ mm}^3/\text{min}, \quad \alpha = \frac{\pi}{6}.$

$$r = h \tan \alpha$$

$$= h \tan \frac{\pi}{6}$$

$$= \frac{h}{\sqrt{3}}.$$

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \left(\frac{h}{\sqrt{3}} \right)^2 h$$

$$= \frac{\pi}{9} h^3.$$

$$\therefore \frac{dV}{dt} = \frac{\pi}{3} h^2 \frac{dh}{dt}.$$

When $h = 10 \text{ mm},$

$$-1 = \frac{\pi}{3} (10)^2 \frac{dh}{dt}$$

$$-1 = \frac{100\pi}{3} \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = -\frac{3}{100\pi} \text{ mm/min.}$$

 $\frac{1}{2}$ m $\frac{1}{2}$ m $\frac{1}{2}$ m $\frac{1}{2}$ m

25.
a)

$$\text{Let } 4x = \sin \theta, \quad \cos \phi = \frac{4}{5}, \quad \sin \phi = \frac{3}{5}.$$

$$y = \sin^{-1}(\sin \theta \cos \phi - \cos \theta \sin \phi)$$

$$= \sin^{-1}[\sin(\theta - \phi)]$$

$$= \theta - \phi.$$

$$\therefore y = \sin^{-1}(4x) - \sin^{-1}\left(\frac{3}{5}\right).$$

$$\frac{dy}{dx} = \frac{d}{dx} \left[\sin^{-1}(4x) - \sin^{-1}\left(\frac{3}{5}\right) \right]$$

$$= \frac{4}{\sqrt{1 - (4x)^2}}$$

$$= \boxed{\frac{4}{\sqrt{1 - 16x^2}}}.$$

1 m

½ m

½ m

OR

$$\sqrt{3}(x^2 + y^2) = 4xy$$

$$2\sqrt{3} \left(x + y \frac{dy}{dx} \right) = 4 \left(x \frac{dy}{dx} + y \right)$$

$$2\sqrt{3}x + 2\sqrt{3}y \frac{dy}{dx} = 4x \frac{dy}{dx} + 4y$$

$$\frac{dy}{dx}(2\sqrt{3}y - 4x) = 4y - 2\sqrt{3}x$$

$$\therefore \frac{dy}{dx} = \frac{4y - 2\sqrt{3}x}{2\sqrt{3}y - 4x} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$$

$$\frac{dy}{dx} = \frac{\sqrt{3} - \frac{\sqrt{3}}{2}}{\frac{3}{2} - 1} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{\sqrt{3}}$$

1 m

1m

26.

Injectivity (One-One):Let $f(x_1) = f(x_2)$.

$$\Rightarrow x_1^2 + x_1 + 2 = x_2^2 + x_2 + 2$$

$$\Rightarrow x_1^2 - x_2^2 + x_1 - x_2 = 0$$

$$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0,$$

$$\therefore x_1 = x_2 \quad \text{or} \quad x_1 + x_2 = -1,$$

For example,

$$f(0) = 2, \quad f(-1) = 2,$$

$$\therefore f(0) = f(-1), \quad 0 \neq -1,$$

$\therefore f$ is not injective (not one-one).

1 m +

 $\frac{1}{2}$ m**Surjectivity (Onto):**

$$f(x) = x^2 + x + 2$$

$$= \left(x + \frac{1}{2}\right)^2 + \frac{7}{4} \geq \frac{7}{4}, \quad \forall x \in \mathbb{R}.$$

$$\therefore \text{Range of } f = \left[\frac{7}{4}, \infty\right) \neq \text{Codomain } \mathbb{R}.$$

$\therefore f$ is not surjective (not onto).

1 m +

 $\frac{1}{2}$ m

27.

$$\int (\sin^{-1} x)^2 dx = \int t^2 \cos t dt \quad (t = \sin^{-1} x, x = \sin t, dx = \cos t dt)$$

$$= t^2 \sin t - \int 2t \sin t dt \quad (\text{Integration by Parts})$$

$$= t^2 \sin t - \left[-2t \cos t + \int 2 \cos t dt\right]$$

$$= t^2 \sin t + 2t \cos t - 2 \sin t + C$$

$$= x(\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C.$$

$$\int (\sin^{-1} x)^2 dx = x(\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C$$

 $\frac{1}{2}$ m

2 m

 $\frac{1}{2}$ m

OR

	$I = \int_0^{\pi} \frac{(\pi - x) dx}{p^2 \sin^2 x + q^2 \cos^2 x}.$ $2I = \pi \int_0^{\pi} \frac{dx}{p^2 \sin^2 x + q^2 \cos^2 x}$ $= 2\pi \int_0^{\pi/2} \frac{dx}{p^2 \sin^2 x + q^2 \cos^2 x}.$ $I = \pi \int_0^{\pi/2} \frac{\sec^2 x dx}{p^2 \tan^2 x + q^2}.$ $u = \tan x, \quad du = \sec^2 x dx.$ $I = \pi \int_0^{\infty} \frac{du}{p^2 u^2 + q^2}$ $= \frac{\pi}{p^2} \left[\frac{1}{q/p} \tan^{-1} \left(\frac{pu}{q} \right) \right]_0^{\infty}$ $= \frac{\pi}{pq} \left(\frac{\pi}{2} - 0 \right).$ $\boxed{I = \frac{\pi^2}{2pq}}$	½m 2 m ½ m
28.	$E = \cot^{-1} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right]$ $= \cot^{-1} \left[\frac{(\sqrt{1+x} - \sqrt{1-x})^2}{(1+x) - (1-x)} \right]$ $= \cot^{-1} \left[\frac{1 - \sqrt{1-x^2}}{x} \right]$ $= \cot^{-1} \left[\frac{x}{1 + \sqrt{1-x^2}} \right]$ $= \cot^{-1} \left[\tan \left(\frac{1}{2} \sin^{-1} x \right) \right]$ $= \frac{\pi}{2} - \frac{1}{2} \sin^{-1} x$ $= \boxed{\frac{\pi}{4} + \frac{1}{2} \cos^{-1} x}.$	½ m ½ m ½ m ½ m ½ m

29	$f(x) = 2x^3 - 24x + 107,$ $f'(x) = 6x^2 - 24 = 6(x - 2)(x + 2).$ $\therefore$ Critical points: $x = \pm 2$. On $[1, 3]$ $f(1) = 85, \quad f(2) = 75, \quad f(3) = 89.$ Maximum = 89 at $x = 3$ On $[-3, -1]$ $f(-3) = 125, \quad f(-2) = 139, \quad f(-1) = 129.$ Maximum = 139 at $x = -2$	1 m 1 m 1m
30.	$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix},$ $5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, \quad 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}.$ $A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O.$ $\therefore A^2 - 5A + 7I = O$	2m
(b)	$A - 5I + 7A^{-1} = O$ $7A^{-1} = 5I - A$ $= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ $= \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}.$ $A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$	2m

31

$$\begin{aligned}\text{LHL: } \lim_{x \rightarrow 0^-} \frac{1 - \cos 4x}{x^2} &= \lim_{x \rightarrow 0^-} \frac{2 \sin^2 2x}{x^2} \\ &= 2(4) \lim_{x \rightarrow 0^-} \left(\frac{\sin 2x}{2x} \right)^2 \\ &= 8.\end{aligned}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{\sqrt{x} (\sqrt{\sqrt{x} + 16} + 4)}{(\sqrt{\sqrt{x} + 16} - 4)(\sqrt{\sqrt{x} + 16} + 4)}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{\sqrt{x} (\sqrt{\sqrt{x} + 16} + 4)}{\sqrt{x} + 16 - 16}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{\sqrt{x} (\sqrt{\sqrt{x} + 16} + 4)}{\sqrt{x}}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} (\sqrt{\sqrt{x} + 16} + 4)$$

$$\text{RHL} = \sqrt{0 + 16} + 4 = 8$$

OR

$$\begin{aligned}y &= (\cos x)^y \\ \log y &= y \log(\cos x).\end{aligned}$$

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \frac{dy}{dx} \log(\cos x) - y \tan x \\ \frac{dy}{dx} \left[\frac{1}{y} - \log(\cos x) \right] &= -y \tan x \\ \frac{dy}{dx} \left[\frac{1 - y \log(\cos x)}{y} \right] &= -y \tan x.\end{aligned}$$

$$\boxed{\frac{dy}{dx} = -\frac{y^2 \tan x}{1 - y \log(\cos x)} = \frac{y^2 \tan x}{y \log(\cos x) - 1}}$$

1½ m

1½ m

½ m

2m

½ m

32.

1. Reflexive: For $a = \frac{1}{2}$, $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

Since $\frac{1}{2} > \frac{1}{4}$, $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$.

$\therefore R$ is NOT reflexive.

2. Symmetric: Take $a = 1$, $b = 2$.

$1 \leq 2^2 = 4 \Rightarrow (1, 2) \in R$.

However, $2 \leq 1^2 = 1$ is false

$\Rightarrow (2, 1) \notin R$.

$\therefore R$ is NOT symmetric.

3. Transitive: Take $a = 3$, $b = -2$, $c = 1$.

$a \leq b^2 \Rightarrow 3 \leq (-2)^2 = 4 \Rightarrow (3, -2) \in R$.

$b \leq c^2 \Rightarrow -2 \leq 1^2 = 1 \Rightarrow (-2, 1) \in R$.

But $a \leq c^2 \Rightarrow 3 \leq 1^2 = 1$ is false

$\Rightarrow (3, 1) \notin R$.

$\therefore R$ is NOT transitive.

1 m

2 m

2 m

33

Let $u = (\sin x)^{x \cos x}$.

$\log u = x \cos x \log(\sin x)$.

$\frac{1}{u} \frac{du}{dx} = \cos x \log(\sin x) - x \sin x \log(\sin x) + x \cos x \frac{\cos x}{\sin x}$.

$\therefore \frac{du}{dx} = (\sin x)^{x \cos x} [\cos x \log(\sin x) - x \sin x \log(\sin x) + x \cot x \cos x]$.

For $v = \sin^{-1}(\sqrt{x})$,

$$\begin{aligned} \frac{dv}{dx} &= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{x(1-x)}}. \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{du}{dx} + \frac{dv}{dx} \\ &= (\sin x)^{x \cos x} [\cos x \log(\sin x) - x \sin x \log(\sin x) + x \cot x \cos x] + \frac{1}{2\sqrt{x(1-x)}}. \end{aligned}$$

OR

 $\frac{1}{2}$ m

2m

1
 $\frac{1}{2}$
m

1m

	$x = a(\cos t + t \sin t), \quad y = a(\sin t - t \cos t)$ $\frac{dx}{dt} = at \cos t,$ $\frac{dy}{dt} = at \sin t.$ $\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t}$ $\frac{d^2y}{dx^2} = \frac{d}{dt}(\tan t) \frac{dt}{dx}$ $= \sec^2 t \left(\frac{1}{at \cos t} \right)$ $= \boxed{\frac{\sec^3 t}{at}}.$	2m 2m 1m
34	1. Determinant of B $ B = 3(12 - 10) + 1(-30 + 25) + 1(30 - 30) = 1$ Thus, $ B = 1$ 2. Cofactor matrix of B $C = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}$ Hence, $B^{-1} = \frac{1}{ B } C^T = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ 3. Find $(BA)^{-1}$ $(BA)^{-1} = A^{-1}B^{-1}$ $= \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ $\boxed{(BA)^{-1} = \begin{bmatrix} 12 & 0 & -7 \\ 13 & 3 & 1 \\ -10 & -1 & 3 \end{bmatrix}}$	½ m 1 m 1½ m 1/2 m ½ m 1 m

35
a)

$$\frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 2}$$

$$x^2 + x + 1 = (Ax + B)(x + 2) + C(x^2 + 1).$$

$$x = -2: \quad 3 = 5C \quad \Rightarrow \quad C = \frac{3}{5},$$

$$A + C = 1 \quad \Rightarrow \quad A = \frac{2}{5},$$

$$2B + C = 1 \quad \Rightarrow \quad B = \frac{1}{5}.$$

$$\int \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} dx = \frac{1}{5} \int \frac{2x}{x^2 + 1} dx + \frac{1}{5} \int \frac{dx}{x^2 + 1} + \frac{3}{5} \int \frac{dx}{x + 2}$$

$$= \boxed{\frac{1}{5} \log(x^2 + 1) + \frac{1}{5} \tan^{-1} x + \frac{3}{5} \log|x + 2| + C}.$$

½ m

3m

1½
m

OR

$$I = \int_0^{\pi/4} \log(1 + \tan x) dx$$

$$= \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$$

$$= \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan x}\right) dx$$

$$= \int_0^{\pi/4} [\log 2 - \log(1 + \tan x)] dx$$

$$= \frac{\pi}{4} \log 2 - I.$$

$$2I = \frac{\pi}{4} \log 2$$

$$\boxed{I = \frac{\pi}{8} \log 2}$$

½ m

3m

1½
m

36

$$(i) \quad \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

Thus, $AX = B.$

$$(ii) \quad |A| = 1 \begin{vmatrix} 3 & 1 \\ -1 & -7 \end{vmatrix} - 1 \begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} - 1 \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix}$$

$$= 1(-21 + 1) - 1(-14 - 3) - 1(-2 - 9)$$

$$= -20 + 17 + 11$$

$$= 8 \neq 0.$$

-2m

	(iii) Cofactors of A : $C_{11} = -20, C_{12} = 17, C_{13} = -11,$ $C_{21} = 8, C_{22} = -4, C_{23} = 4,$ $C_{31} = 4, C_{32} = -3, C_{33} = 1.$ $\text{adj}(A) = \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \text{adj}(A) = \frac{1}{8} \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$ -2m
37.	(i) $f : \mathbb{N} \rightarrow \mathbb{R}, f(x) = x^2, x \in \{1, 2, 3, \dots\}$ $\therefore \text{Range}(f) = \{1, 4, 9, 16, 25, \dots\}$ $= \{x^2 : x \in \mathbb{N}\}.$ (ii) $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2$ $\Rightarrow x_1 = x_2 \quad (x_1, x_2 \in \mathbb{N}).$ $\therefore f \text{ is injective (one-one).}$ iii) $f(x) = x^2 + x + 2$ $\mathbb{R}$ to $\mathbb{R}$ -Proving: Neither injective nor bijective To check if $f(x) = \frac{x}{\sqrt{1+x^2}}$ is one-to-one, assume $f(a) = f(b)$: PNG $\frac{a}{\sqrt{1+a^2}} = \frac{b}{\sqrt{1+b^2}}$ - Since $\sqrt{1+x^2} > 0$, a and b must have the same sign. - Square both sides: $\frac{a^2}{1+a^2} = \frac{b^2}{1+b^2} \implies a^2(1+b^2) = b^2(1+a^2) \implies a^2 = b^2$ - Taking the square root gives $a = \pm b$. Since a and b share the same sign, $a = b$. Conclusion: f is one-to-one. PNG 1 m 1m 1+1 2m

38.	(i) Subscribers discontinuing = $10x$. Remaining subscribers = $5000 - 10x$. (ii) $R(x) = (5000 - 10x)(300 + x)$ $= 1,500,000 + 2000x - 10x^2$. (iii)(a) $R'(x) = 2000 - 20x$. $R'(x) = 0 \Rightarrow 2000 - 20x = 0$ $\Rightarrow x = 100$. $R''(x) = -20 < 0$. $\therefore$ Revenue is maximum when $x = \text{Rs } 100$. OR $R'(x) = 2000 - 20x = 20(100 - x)$. $x \in (0, 100) \Rightarrow R'(x) > 0 \Rightarrow R(x)$ is increasing. $x \in (100, 5000) \Rightarrow R'(x) < 0 \Rightarrow R(x)$ is decreasing.	1 m 1m 1+1 2m
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